Implicaciones de las Nuevas Reglas de Inversión de la CONSAR para la Construcción de Portafolios Óptimos y el Impacto sobre la Garantía de Pensión Mínima

Implications of CONSAR´s New Investment Rules for the Construction of Optimal Portfolios and the Impact on the Minimum Pension Guarantee

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RESEÑA

1. Objetivo

El objetivo del presente trabajo es comparar el nuevo régimen de inversión con las reglas anteriores en términos de la probabilidad de ejercer la garantía de pensión mínima (GPM). Se busca determinar si hay una diferencia estadística significativa entre las probabilidades de ejercer dicha garantía. Este análisis se hace para trabajadores con distintos niveles de salarios y con un cargo de comisión promedio. Asimismo, se considera la carrera salarial del trabajador promedio para estimar la probabilidad de ejercer la GPM bajo las reglas del nuevo régimen de inversión. Por último, también bajo el nuevo régimen, se analiza si existe una diferencia estadística significativa entre las probabilidades de ejercer la GPM cuando se usan los dos niveles de comisiones más baratos y los dos más caros.

2. Planteamiento

En México, el sistema de cuentas individuales de retiro para trabajadores del sector privado ha sido objeto de cambios importantes en los últimos meses. Durante 2007 (año del décimo aniversario del sistema de cuentas individuales), tanto el régimen de inversión como la estructura de comisiones fueron modificados.

El anterior régimen de inversión solamente tenía disponibles dos portafolios de inversión para las AFORES y los trabajadores eran asignados a uno de estos portafolios en base a consideraciones de ciclo de vida. Los trabajadores viejos eran asignados al portafolio que solo podía invertir en instrumentos de renta fija y requería de un mínimo de 51% de protección inflacionaria. Por su parte, los trabajadores jóvenes podían escoger el portafolio que permitía hasta un 15% en instrumentos de renta variable. Los cambios recientes al régimen de inversión

crean tres portafolios adicionales con límites de 30%, 25% y 20% en instrumentos de renta variable, respectivamente. Los trabajadores continuarán siendo asignados a uno de los cinco portafolios también en base a consideraciones de ciclo de vida.

Por lo tanto, es de gran relevancia estudiar las implicaciones del nuevo régimen de inversión para los saldos finales de las cuentas individuales de retiro. El monto final en dichas cuentas determinará si el trabajador recurre o no a su derecho de ejercer la garantía de pensión mínima. En la medida en que la probabilidad de ejercer dicha garantía sea estadísticamente significativamente menor que la correspondiente al anterior régimen, entonces se podrá evaluar positivamente al nuevo régimen de inversión desde esta perspectiva.

3. Hipótesis

En este trabajo identificaremos si la probabilidad de ejercer la garantía de pensión mínima (GPM) bajo el nuevo régimen de inversión es estadísticamente significativamente menor que la correspondiente al anterior régimen. Estudios previos solo han estimado dicha probabilidad en un contexto de una proporción fija exógena (no óptima) de instrumentos de renta variable dentro del portafolio. Por esta misma razón, dichos estudios no hubieran podido contrastar el nuevo régimen de inversión con el anterior en términos de sus efectos sobre la probabilidad de ejercer la garantía ya mencionada. Como en este estudio los portafolios forman parte de la frontera eficiente y son reconstruidos mes a mes, esto nos permite investigar si los mayores límites de inversión en renta variable (léase el nuevo régimen) tienen efectos estadísticamente significativamente diferentes sobre la probabilidad de ejercer la GPM. Las pruebas de hipótesis las hacemos para trabajadores con distintos niveles de salarios y con un cargo de comisión promedio. En este trabajo también estimamos la probabilidad de ejercer la GPM cuando quitamos el supuesto poco realista de salarios constantes durante la vida laboral y, en su lugar, consideramos la carrera salarial del trabajador promedio. Por último, también analizamos el papel que juega el nivel de las comisiones en la probabilidad de ejercer la GPM.

4. Metodología

La metodología del presente trabajo se divide en tres partes. En la primera parte hacemos uso de varias herramientas para la simulación del proceso de acumulación de saldos de las cuentas individuales de retiro.

1. Obtenemos una distribución normal multivariante de los rendimientos mensuales de los activos permitidos por la CONSAR.
2. Construimos portafolios óptimos basándonos en la matriz de varianza-covarianza generada en el paso 1 y usando el algoritmo de la línea crítica.

3. Generamos rendimientos aleatorios de la distribución normal multivariante.
4. Obtenemos el rendimiento del portafolio de los pasos 2 y 3.
5. Aplicamos el rendimiento del portafolio al saldo de la cuenta individual de retiro. También se cobra la comisión sobre saldo cada doce meses.
6. Eliminamos los primeros rendimientos mensuales e incorporamos los del paso 3 como los últimos.
7. Repetimos los pasos del 1 al 6 hasta completar 300 meses (25 años) de cotización.

En la segunda parte estimamos la probabilidad de ejercer la GPM después de haber hecho mil simulaciones de acumulación de saldos. Dicha probabilidad se obtiene al dividir el número de saldos finales que fueron menores al monto constitutivo de la garantía de pensión mínima.

En la tercera y última etapa hacemos pruebas de hipótesis para determinar si hay diferencia estadística significativa entre las probabilidades de ejercer la GPM bajo ya sea distintos regímenes de inversión o distintos niveles de comisiones.

5. Conclusiones

Durante 2007, importantes modificaciones fueran hechas al régimen de inversión para incrementar la diversificación de portafolio y, posiblemente, para obtener mayores rendimientos al incrementar los límites de inversión en instrumentos de renta variable. Si particularmente esto último pudiera lograrse, habría efectos favorables sobre los saldos finales de las cuentas individuales de retiro y, por lo tanto, sobre la probabilidad de ejercer la garantía de pensión mínima.

Mediante un proceso sistemático de optimización de portafolio en donde se obtiene la frontera eficiente, nuestros resultados muestran que las AFORES podrían aprovechar el nuevo régimen de inversión para incrementar los saldos finales de las cuentas individuales de retiro. Este hallazgo es particularmente relevante para la probabilidad de ejercer la GPM correspondiente a los trabajadores de bajos niveles salariales (1 y 2 salarios mínimos). Las pruebas de hipótesis muestran que tal probabilidad es estadísticamente significativamente menor (al 95% de confianza) que la obtenida bajo el anterior régimen. Además, cuando nos movemos del portafolio de mínima varianza al de máximo rendimiento a lo largo de la frontera eficiente, la probabilidad de ejercer la GPM disminuye drásticamente para los trabajadores de bajos niveles salariales.

Cuando analizamos a los trabajadores que ganan por lo menos cuatro salarios mínimos, la probabilidad de ejercer la GPM es muy cercana a cero o nula, independientemente del punto

sobre la frontera eficiente en que se esté. Similarmente, cuando consideramos el perfil de carrera salarial del trabajador promedio, dicha probabilidad es cero o casi nula. Finalmente, las diferencias en el nivel de comisiones entre los grupos de AFORES más caras y más baratas solamente tienen un efecto significativamente estadístico sobre aquellos trabajadores que perciben un salario mínimo.

Es importante mencionar que nuestros resultados dependen de un proceso de optimización de portafolio mensual que harían los administradores de fondos de pensiones. Dicho proceso parte del supuesto de que la distribución normal multivariante de los rendimientos de los activos se mantendrá en el futuro. No obstante, esperamos que este trabajo resalte la importancia de la administración activa y eficiente de portafolio para aprovechar al máximo el nuevo régimen de inversión.

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Abstract. This paper compares the new investment regime with the old asset allocation rules in terms of the probability of exercising the Minimum Pension Guarantee (MPG). In an environment of mean-variance efficient portfolios which are re-optimized every month, the final balance of individual retirement accounts is obtained by an accumulation process which uses the returns of such portfolios for a given commission and salary level. The lifetime wage profile of an average worker is also considered in this analysis. Our results suggest that 1) the probability of exercising the MPG considerably decreases as we move from the minimum-variance portfolio to the maximum-return portfolio along the efficient frontier; 2) only for workers earning between one and two minimum wages, the probability of exercising the MPG under the new investment rules is statistically lower than that corresponding to the old investment regime for most of the portfolios along the efficient frontier; 3) for workers earning at least four minimum wages, such probability is either close to zero or nil regardless of the point on the efficient frontier; 4) the probability of resorting to the MPG is almost zero or nil when the life-cycle salary profile for the average worker is used in the accumulation of pension balances; and 5) for workers earning just one minimum wage, there is statistical difference between the probabilities of exercising the MPG corresponding to the groups of the cheapest and most expensive commissions, but not for the difference between the probabilities corresponding to both groups for other wage levels and the life-cycle salary profile.

JEL Classification: C14, G11, G23
Keywords: Pension funds, efficient frontier, minimum pension guarantee.

1. Introduction

The pension system for workers of the formal private sector in Mexico has gone through important changes in recent months. In 2007 (on the 10th anniversary of the system of individual accounts) both the investment regime and the structure of commissions were modified.

The previous investment regime made only two portfolios available for each pension fund administrator (AFORE) and workers were assigned to one of these portfolios based on life-cycle considerations. Older workers were assigned to a portfolio that could only invest in fixed income instruments that required a minimum of 51% of assets with inflationary protection. Younger workers could choose the portfolio that allowed up to 15% in variable income securities. Recent changes introduced three additional portfolios (*SIEFORES Básicas* 3, 4, and 5) with maximum limits of 20%, 25% and 30% on equity investment, respectively.[1] Workers will continue to be assigned to the new portfolios based on life-cycle considerations.

With respect to commissions, AFORES used to charge two types of commissions, one on the account balance and another on contributions (inflows) to the individual account. The 2007 reform will allow AFORES to only charge a commission on account balances.

This paper compares the new investment regime with the old asset allocation rules in terms of the probability of exercising the Minimum Pension Guarantee (MPG). We use a model of portfolio optimization proposed by Markowitz (1959) to obtain the mean-variance efficient frontier of portfolios. In such a context we re-optimize the portfolios every month and the final balance of individual retirement accounts is obtained by an accumulation process which uses the returns of such portfolios for a given commission fee and salary level. The lifetime wage profile of an average worker is also considered in this analysis.

Our results suggest that 1) the probability of exercising the MPG considerably decreases as we move from the minimum-variance portfolio to the maximum-return portfolio along the efficient frontier; 2) only for workers earning between one and two minimum wages, the probability of exercising the MPG under the new investment rules is statistically lower than that corresponding to the old investment regime for most of the portfolios along the efficient frontier; 3) for workers earning at least four minimum wages, such probability is either close to

[1] Equity investment is only allowed through stock market indices, not through individual stocks. *Circular CONSAR 15-19, Diario Oficial de la Federación de Julio 9 de 2007.*

zero or nil regardless of the point on the efficient frontier; 4) the probability of resorting to the MPG is almost zero or nil when the life-cycle salary profile for the average worker is used in the accumulation of pension balances; and 5) for workers earning just one minimum wage, there is statistical difference between the probabilities of exercising the MPG corresponding to the groups of the cheapest and most expensive AFORES (in terms of commissions), but there is no statistical difference between the probabilities corresponding to both groups of AFORES for higher wage levels, and when taking into account the life-cycle salary profile.

In the next section we describe the minimum pension guarantee. Section 3 summarizes the changes that the investment regime for AFORES has gone through. Section 4 presents a model of accumulation of pension fund balances. Section 5 shows how to estimate the probability of exercising the MPG. Section 6 explains the general mean-variance problem. Section 7 provides the solution to the general mean-variance problem. Section 8 applies the general mean-variance problem to Mexican pension funds. Section 9 contains the simulation results of the general mean-variance problem applied to Mexican pension funds. Finally, Section 10 presents our concluding remarks.

2. The Minimum Pension Guarantee

The Social Security Law, in effect since 1997, establishes a Minimum Pension Guarantee (MPG) for those workers whose final individual account balances are insufficient to buy an annuity equivalent to a minimum wage (adjusted for inflation). To be eligible to receive the MPG, workers could fall into either one of two possible categories:[2]

- Unemployment in Old Age (*Cesantía en Edad Avanzada*). This category applies to those workers who were left unemployed at 60 years old or older.
- Retirement (*Retiro por Vejez*). This category is for workers who want to retire at 65 years of age or older.

It is important to stress that under the current law workers are eligible for the minimum pension guarantee only if they have contributed a minimum of 1,250 weeks (equivalent to 24 years).

The procedure for exercising the MPG is as follows:

[2] Article 170 of the Law of Social Security published in 1995, but in effect since July 1997.

i) When an insured worker wants to retire, the Mexican Institute of Social Security (IMSS) has the obligation to calculate an approximate monthly pension which is based on the final balance of her retirement account.

ii) If the worker contributed at least once prior to July 1997, the worker has the option to retire under the old law that established a defined benefit pension.[3]

iii) If the worker decides to retire under the new regime she has two options:

   a. To buy an annuity from an insurance company: under this alternative the insurance company bears the risk of survival at old age.
   b. Define a monthly withdrawal with the worker's AFORE: under this alternative the worker assumes the risk of living beyond her last withdrawal.

iv) If the worker's account balance is insufficient to buy either an annuity or a defined withdrawal equivalent to one minimum wage (when she has contributed for at least 1,250 weeks), the Federal Government will contribute additional resources to guarantee a monthly pension equivalent to the minimum wage adjusted for inflation and the acquisition of life insurance for the worker's beneficiaries.[4]

v) Resources from the individual account would have to be exhausted prior to the government's contribution and at this point the IMSS would be in charge of paying the monthly pension to the worker or her beneficiaries.

vi) When a retired worker receiving the minimum pension guarantee dies, the Federal Government will buy an annuity from an insurance company (chosen by the beneficiaries) to pay the following benefits:

   a. Ninety percent of the minimum pension goes to the surviving spouse.
   b. Twenty percent of the minimum pension goes to any of the surviving descendants or thirty percent when both insured parents die.[5]
   c. In the absence of spouse and descendants, twenty percent of the minimum pension would be paid to each of the worker's surviving parents.

The determinants for exercising the Minimum Pension Guarantee come from worker-specific characteristics (such as salary, contribution rate, age, and working lifespan) as well as from pension fund administrator-specific characteristics (commission fees and investment returns).

[3] The worker would still have to meet the requirements for retirement under the old law.

[4] If the worker's individual account balance is insufficient but has not contributed to the system for at least 1,250 weeks, the worker has the option of working additional time to meet this requirement or withdraw her account balance in a lump-sum payment.

[5] The benefits a and b are non-mutually exclusive. In other words, a surviving spouse and any of the surviving descendants would get 90% and 20% of the minimum pension guarantee, respectively.

In this paper we will compare investment returns under the new and the old investment regime for a given set of worker-specific characteristics and AFORE's commission fees.

3. The Investment Regime

Individual accounts are administered by AFORES and invested by specialized investment subsidiary entities called SIEFORES (*Sociedades de Inversión Especializadas en Fondos de Ahorro para el Retiro*). When a worker chooses (or is assigned to) a particular AFORE, the funds in the individual account are invested by that AFORE's SIEFORE.

SIEFORES are subject to an investment regime determined by the regulatory authority, CONSAR (*Comisión Nacional del Sistema de Ahorro para el Retiro*), which among other things sets limits to the risk that each SIEFORE can take on with its investment choices. The investment regime has been gradually modified to allow greater diversification while maintaining risk levels within certain limits.

Between 2002 and 2004 important changes were introduced to the investment regime. These changes were related to the following:

1) Since 1997 a minimum of 65% of total assets had to be invested in government bonds with a maximum maturity (or revision) period of 183 days.[6] This restriction was first modified through an increase in the average maturity period to 900 days (December 2001).[7] Subsequently, the regulation was based on a daily valuation of risk (a VaR measure) in effect since November 2002.[8]
2) Originally, the type of issuer was restricted to the following limits:[9]
    a. Federal Government (minimum 65%)
    b. Private Issuers (maximum 35%)
    c. Financial Intermediaries (maximum 10%)

    These limits were removed and replaced by creditworthiness (with a minimum rating of A) as measured by credit rating agencies. In addition, instruments issued by local and state governments and government-owned companies were allowed. [10]
3) The use of derivatives was introduced (November 2002). [11]

[6] *Circular CONSAR 15-1, Diario Oficial de la Federación de Junio 30 de 1997.*
[7] *Circular CONSAR 15-5, Diario Oficial de la Federación de Diciembre 5 de 2001.*
[8] The VaR limit was set at 0.6% over total net assets. *Circular CONSAR 15-8, Diario Oficial de la Federación de Noviembre 29 de 2002.*
[9] *Circular CONSAR 15-1.*
[10] *Circular CONSAR 15-6, Diario Oficial de la Federación de Abril 8 de 2002.*

4) Investment in equity was allowed up to 15% of total assets, as long as they tracked stock indices (May 2004).[12]
5) Exclusion of foreign issuers was lifted and their portfolio share could not exceed 20% (this share includes both bonds and stocks) as long as the issuers were under the regulation of the technical committee of the International Organization of Securities Commissions (IOSCO) and/or the European Union (May 2004).[13]
6) In May 2004 two investment funds were created based on life-cycle considerations:
    a. SIEFORE *Básica* 1 for workers with at least 56 years of age. This fund can only be invested in bonds (domestic and foreign (up to 20%)) and at least 51% is required to have some sort of inflationary protection.[14]
    b. SIEFORE *Básica* 2 for workers under 56 years of age. This fund allows investment in stocks (up to 15%) and up to 20% in foreign instruments (bonds or stocks).[15]

In July 2007 a new set of changes was introduced to the investment regime, although the changes will take effect in March 2008. CONSAR approved the inclusion of three additional investment funds: *SIEFORES Básicas* (SBs) 3, 4 and 5, with the allocation of workers based on life-cycle considerations. In all, there will be five investment funds aimed at workers of different ages: SB5 for workers between 18 and less than 27 years old, SB4 for workers between 27 and less than 37 years old, SB3 for workers between 37 and less than 46 years old, SB2 for workers between 46 and less than 56 years old, and SB1 for workers of 56 years and older.

The new investment regime allows SIEFORES 2, 3, 4 and 5 to invest in *Notas*[16], variable income instruments, and Real Estate and Infrastructure Trusts, REITs (*Fideicomisos de Infraestructura y Bienes Raíces, FIBRAs*). The new instruments aim at increasing portfolio diversification and obtaining higher returns, at the same time that more agents gain access to new sources of finance.

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[11] *Circular CONSAR 15-8.*
[12] *Circular CONSAR 15-12, Diario Oficial de la Federación de Mayo 26 de 2004.*
[13] Ibid.
[14] These instruments can either be denominated in "Unidades de Inversión, UDIs" (inflation-indexed units) or must guarantee a return equal or greater than the variation in UDIs.
[15] The VaR limit for SIEFORE *Básica* 2 was set at 1% of total net assets.
[16] Domestic and foreign debt instruments with the principal protected until maturity and linked to one or more of the permitted equity indices.

With respect to prudential regulation of risk, limits based on VaR measures were established for each of the SIEFORES.[17] AFORES could take advantage of these differences in order to differentiate themselves from the rest in terms of returns. An additional measure in terms of risk concerns the losses attributable to AFORES' mismanagement of pension funds and the reserves that they have to maintain to cover these losses. The regime for calculating attributable losses conforms to international best practices.

The recent modifications to the investment regime reflect the desirability of obtaining higher returns while maintaining workers' savings within acceptable risk limits. It is important to mention that all AFORES have expressed their interest in offering the whole range of investment funds now allowed and this could result in increased sophistication of pension fund managers in the administration of resources.

By taking advantage of new methods of securitization in the domestic capital market, the new investment regime increases the limits on investment in structured securities and securitized instruments that satisfy certain conditions.[18] It is important to mention that securitizations have the characteristic that the issuance of the instrument does not represent a credit risk linked to the originator, but linked directly to the underlying portfolio of securitized collection rights.

Finally, by allowing Real Estate and Infrastructure Trusts, REITs, on the list of permitted instruments, access to finance for a wider range of agents is facilitated along with some deepening of the domestic financial system. REITs instruments make the undertaking of infrastructure investment projects possible, which in turn helps AFORES extend their investment horizon while earning higher returns.

The following table summarizes the main characteristics of the new investment regime to take effect in March 2008.

[17] VaR limits were established considering that those workers closer to the age of retirement should face lower risks.
[18] These conditions are specified in the appendices to Circular CONSAR 15-19. In particular, appendix K and appendix L.

Summary of the New Investment Regime

			Limits by kind of SIEFORE[1]				
			1	**2**	**3**	**4**	**5**
Market Risk	Value at Risk [$VaR_{historical}$ (1-Ξ□= 95%, 1 day)]		0.6%	1.0%	1.3%	1.6%	2.0%
	Equity (only through indices)		0%	15%	20%	25%	30%
	Foreign Currency (Dollars, Euros, Yenes or curencies to acquire equity)		30%	30%	30%	30%	30%
	Derivatives		YES	YES	YES	YES	YES
Credit Risk	mxAAA[2] and Government Securities		100%	100%	100%	100%	100%
	mxAA- rated securities		35%	35%	35%	35%	35%
	mxA- rated securities		5%	5%	5%	5%	5%
Concentration/ Counterparty Risk	**Local Securities**	mxAAA rated securities from one issuer[3] or counterpart	5%	5%	5%	5%	5%
		mxAA rated securities from one issuer or counterpart	3%	3%	3%	3%	3%
		mxA rated securities from one issuer or counterpart	1%	1%	1%	1%	1%
		BBB+ rated securities denominated in foreign currency from one issuer	5%	5%	5%	5%	5%
		BBB- rated securities denominated in foreign currency from one issuer	3%	3%	3%	3%	3%
	Foreign	A- rated foreign securities from one issuer or counterpart	5%	5%	5%	5%	5%
		Maximum ownership of one issue[4]	20%	20%	20%	20%	20%
Other Limits	Foreign securities (if fixed income, minimum rate is A-)		20%	20%	20%	20%	20%
	Securitizations[5]		10%	15%	20%	30%	40%
	Structured securities (issued by Mexican nationals)[5]		0%	1%	5%	7.5%	10%
	REITs (Real estate must be in Mexican territory)		0%	5%	5%	10%	10%
	Inflation protected securities minimum[6]		51%	NO	NO	NO	NO
Conflict of Interest	Securities endorsed by related parties		15%	15%	15%	15%	15%
	Securities endorsed by parties related to the Afore[7]		5%	5%	5%	5%	5%

1.- All limits expressed as percentages of assets under management (Activos Netos) but the maximum ownership of one issue. All limits are maximums, except for the inflation protected securities minimum.

2.- These are local rates for securities issued by Mexican nationals in Mexico. Global rates apply to foreign securities All securities must have at least two rates. All securities must be issued through public offers.

3.- Issuer or endorser in the percentage it guarantees. Counterparty exposure in repos and derivatives must be added to exposure acquired through securities.

4.- Percentage of the total amount stated in the prospectus, adjusted by later amortizations and repurchases.

5.- Securitizations must comply with Circular 15-19's appendix K to consider the SPV as an independent issuer; structured securities must comply with appendix L.

6.- Minimum limit on securities that guarantee a return equal or in excess of Mexico's inflation rate.

7.- This limit is contained in the Law (art. 48, fracc. X), although Circular 15-19 sets a limit of 0% for related parties that are financial institutions.

4. Accumulation Model of Pension Fund Balances

In this section we explain the accumulation process of contributions to individual retirement accounts. Such accumulation of funds depends on variables like the base salary of contribution, the commissions' structure, portfolio returns and the years of contributions to these accounts.

The Mexican Law of Social Security, in effect since 1997, states that the administration of savings for retirement requires opening an individual account in one Administrator of Funds for Retirement or AFORE (*Administradora de Fondos para el Retiro*). This account consists of contributions made by the worker, the employer and the government. An individual retirement account consists of three components: a) retirement and unemployment at old age; b) housing; and c) voluntary contributions.

Our analysis only focuses on the accumulation of balances for the retirement and unemployment at old age component. Consequently, the housing component is not considered for the computational simulations for two reasons: 1) this component is administered by the *Instituto del Fondo Nacional para la Vivienda de los Trabajadores* (INFONAVIT) and not by AFORES and 2) because it is not necessarily part of the pension income received during retirement. This is the case since a worker could always resort to the housing component to either pay for a loan made by INFONAVIT or for any other purpose established by this institution. Likewise, voluntary contributions are not taken into account for the simulations done in this work.

The retirement and unemployment at old age component basically receives two types of contributions: 1) the worker, the employer and the government chip in 6.5% of the base salary of contribution;[19] and 2) the social quota, which is a constant real contribution made by the government.[20]

The base salary of contribution refers to the number of earned minimum wages used to calculate the monthly contribution to an individual account. We used several levels for the base salary of contribution by using multiples of the minimum wage. We set such wage to $48.88 pesos, which was the official general minimum wage in December of 2007. Such levels are 1,

[19] The article 168 of the Law of Social Security establishes the following mandatory contribution percentages as a provision for unemployment at old age: 1.125%, 3.15% and 0.225% of the base salary of contribution comes from the worker, the employer and the government, respectively. Additionally, the employer must make a contribution of 2% to fund retirement.

[20] The social quota represents a contribution made by the government equivalent to 5.5% of the earned minimum wage in the *Distrito Federal* in 1997. Such quota is quarterly adjusted to reflect the inflation rate. *Capítulo sexto, Quinta Sección, Art. 168 de la Ley del Seguro Social.*

2, 3, 4, 5, 10, 15, and 25 times the minimum wage mentioned above. It is worth noting that we assumed constant real wages through time.

Not only the accumulation of balances depends on contributions, but also on commissions. They were proxied by CONSAR's equivalent commissions on balances charged by several AFORES and the system's average commission. According to the *Ley de los Sistemas de Ahorro para el Retiro,* equivalent commissions allow comparisons among AFORES because they use the same base (balances) for various time horizons of contributions.[21]

One of the challenges of this work has to do with the lack of data about the new commissions to be charged on balances as of March 2008. Moreover, there is uncertainty about the evolution of such commissions through time. Consequently, we decided to use the two cheapest and the two most expensive commissions from December 2007. These commission fees are based on a continuous contribution to the same AFORE for 25 years and represent average commission fees during that length of time.

Finally, we assume that the balances of an individual account are invested in foreign and domestic securities, as well as a default risk-free index.[22] The computational simulations to accumulate balances start with a 24 year-old worker who retires after 25 years (300 months) of contributions to her individual account.[23] In each month, we do a portfolio optimization using the Markowitz (1959) method for seventy assets.

As it will be explained in Section 8, the optimization problem is solved subject to either the old or the new CONSAR's investment regime. The latter will be effective as of March 2008. The optimal weights are obtained before knowing the new monthly returns randomly taken from a multivariate normal distribution. Then we proceed to calculate the portfolio return by multiplying the optimal weights by the new monthly returns. Every twelve months the commission on balances is charged.

[21] CONSAR uses an average salary and assumes an amount for initial balances among other assumptions to calculate equivalent commissions. The time horizon of contributions used by CONSAR are 1, 2, 3, 4, 5, 10, 15 and 25 years. To each time horizon corresponds an equivalent commission.

[22] See Appendix A to see the specific set of assets used in this work. Such set is part of a bigger set of investments allowed by CONSAR. It was not possible to use all assets due to an insufficient number of observations for those left out.

[23] According to the *Informe al Ejecutivo Federal y al Congreso de la Unión sobre la Situación Financiera y los Riesgos del Instituto Mexicano del Seguro Social, 2006-2007*, the average age of entrance into the contribution system (or into the *Instituto Mexicano del Seguro Social*) is 24.15 years.

When incorporating all the information presented above, we can represent the accumulation of balances corresponding to an individual retirement account with the following formula:

$$B_{t+1} = \left(e^{\sum_{j=1}^{j=J} w_{j,t}^{*} r_{j,t}} B_t + (0.065) NMW + sq \right) - com_t \qquad (1)$$

where

t is the period $t = (0 \ldots 300)$,

J is the total number of assets,

B_t is the accumulated balance at period t,

$w_{j,t}^{*}$ is the optimal weight for asset i at period t,

$r_{j,t}$ is the return of asset i at period t,

NMW is the number of minimum wages,

sq is the social quota,

com_t is the commission charged on balances every twelve months.

The first term on the right-hand side of the equation refers to the weighted return on balances. The weights are obtained with the Markowitz's (1959) portfolio optimization method. The second and third terms refer to the total mandatory contribution, representing 6.5% of the base salary, and the social quota provided by the government, respectively. It is important to mention that the simulation takes into account the commission charged on balances at the end of the year.

Using equation (1) and a time horizon of twenty-five years, we run one-thousand simulations for each one of the levels for the base salary of contribution. It is worth mentioning that all results are given in real terms since we assume no inflation.

The steps we follow for running one simulation are:

1. We obtain a multivariate normal distribution from a fixed-size rolling window of one hundred and twenty-nine monthly returns. For period 1 (first month of contributions) we use historical returns only.

2. We construct optimal portfolios based on the variance-covariance matrix derived from the fixed-size rolling window from step 1. Such portfolios result from applying the critical line algorithm proposed by Markowitz (1959).
3. We randomly generate returns from the multivariate normal distribution.
4. We obtain the portfolio return from the outcomes in steps 2 (optimal weights) and 3 (random returns).
5. We apply the portfolio return to pension balances. We also apply commissions to these balances every twelve months.
6. We eliminate the oldest returns and incorporate the newest returns from step 3. This allows us to keep the size (129) of the rolling window fixed.
7. We repeat steps 1 through 6 until completing three-hundred months (or 25 years).

5. Estimating the probability of exercising the Minimum Pension Guarantee

The Mexican Law of Social Security (1995) defines a guaranteed pension as that provided by the government for those workers that comply with a series of requirements.[24] According to Rentería (2007), ensuring that all workers have a minimum level of income during retirement is the main reason for the provision of a guaranteed pension. The aforementioned law states that the monthly Minimum Pension Guarantee (MPG) must be equivalent to the real minimum wage for the Distrito Federal in 1997. For December of 2007, we came up with $354,507.47 pesos as the total amount required to provide for the income stream corresponding to the MPG. We also use the minimum wage and the aforementioned equivalent commissions on balances corresponding to the same date.

One-thousand simulations allow us to calculate the probability of exercising the option of resorting to the Minimum Pension Guarantee provided by the government. We do this for the different multiples of the real minimum wage previously mentioned. The probability of exercising this option can be stated as:

$$P_{NMW} = \left[\frac{\sum_{i=1}^{N} I\left[B_{i,T,NMW} < MPG\right]}{N} \right] \qquad (2)$$

[24] One of the established requirements is that the worker reaches 65 years of age with 1250 weekly contributions recognized by the *Instituto Mexicano del Seguro Social* (IMSS). If the worker is older than 65 years and does not have the required number of contributions, she can withdraw the total amount of her individual account or continue working until she reaches the total number of weekly contributions. *Capítulo Sexto, Segunda y Sexta Sección Artículos 154 y 162, Ley del Seguro Social.*

where:

I ≡ an indicator function; N ≡ the total number of simulations made $(N = 1{,}000)$; P_{NMW} ≡ the probability of exercising the MPG for a given base salary; NMW ≡ the base salary or the number of real minimum wages; $B_{i,T,NMW}$ ≡ the accumulated balance of an individual account for simulation i at the time of retirement (T = 25 years); MPG ≡ value of the Minimum Pension Guarantee.

A clarification note regarding the use of the probability formula is in order. After obtaining the final balance of the individual account for simulation *i*, we compare if that amount is smaller $(N_i = 1)$ or greater $(N_i = 0)$ than the MPG value. Then, we add up the simulations where the final balance is not enough to reach the MPG number and divide it by the total number simulations (*N*).

6. The General Mean-Variance Problem

We use the Markowitz's (1959) portfolio optimization model to obtain the mean-variance efficient frontier of portfolios. According to Jacobs *et al* (2005), an efficient mean-variance portfolio can be defined as the expected return and variance portfolio combination (E_p^*, V_p^*) that is not dominated by other feasible pair (E_p, V_p). That is, it has a higher expected return $(E_p^* > E_p)$ but no higher variance $(V_p^* \leq V_p)$; or it has a lower variance $(V_p^* < V_p)$ but no lower expected return $(E_p^* \geq E_p)$. In other words, this efficient mean-variance portfolio provides minimum variance among feasible portfolios for a given expected return or maximum expected return for a given variance.

In order to obtain the mean-variance efficient portfolio, we use the critical line algorithm in Markowitz (1959) to trace the linear set of efficient portfolios. Such algorithm maximizes the expected return subject to a portfolio variance and some set of linear constraints. The latter reflects investment restrictions on the shares allowed in the portfolio for some particular securities.

We consider *n* securities subject to a set of constraints and no short-sales of assets are allowed. The weights corresponding to the different securities are denoted by:

$$\mathbf{X_j} = [X_1, X_2, \ldots X_n]' \qquad (3)$$

According to Markowitz (1959), the expected return on the portfolio is given by the sum of the expected returns on the *n* securities multiplied by their weight. Therefore, the expression can be written as:

$$E_p = \boldsymbol{\mu}_\mathbf{j}' \mathbf{X}_\mathbf{j}, \tag{4}$$

where $\boldsymbol{\mu}_\mathbf{j}$ represents the expected return *n x 1* vector, which is given by the mean of historical returns. In addition, the variance of return on the portfolio is given by:

$$V_p = \mathbf{X}_\mathbf{j}' \mathbf{C} \mathbf{X}_\mathbf{j}, \tag{5}$$

where the term $\mathbf{C}$ represents the covariance matrix.

Moreover, Markowitz (1959) assumes that weights are subject to a set of constraints that take the following form:

$$\mathbf{A}\mathbf{X}_\mathbf{j} = \mathbf{b}, \tag{6}$$

where $\mathbf{A}$ is an *m x n* constraint matrix; $\mathbf{b}$ is an *m* component "right hand side" vector.[25] Furthermore, the sum of asset weights must always be equal to one $\left(\sum X_j = 1\right)$.

7. Solution to the General Problem

The general problem takes the following form:

Maximize E_p

subject to $\mathbf{A}\mathbf{X}_\mathbf{j} = \mathbf{b}$,

$\sum X_j = 1$,

$X_j \geq 0$,

$V_p = \mathbf{X}_\mathbf{j}' \mathbf{C} \mathbf{X}_\mathbf{j}$.

[25] In equation (6) we could have standard individual asset weight restrictions like lower bounds and/or upper bounds as well as specific "current income" restrictions.

In order to solve this optimization problem, we use the critical line algorithm proposed by Markowitz (1959). According to Lewis (1988), this algorithm allows us to generate a one-period portfolio optimization when investor preferences depend on the mean and variance of returns. In particular, the algorithm helps us find corner portfolios.[26] Then, the critical line is formed by taking a linear combination of two corner portfolios.

To solve this problem, a Lagrangian expression is used to maximize E_p subject to a set of constraints and after some matrix algebra work, the critical line can be stated as:

$$\widetilde{\mathbf{M}}\begin{bmatrix} X_1 \\ \vdots \\ X_n \\ \lambda_1 \\ \vdots \\ \lambda_m \end{bmatrix} = \mathbf{R} + \widetilde{\mathbf{S}}\lambda_E, \tag{7}$$

where $\widetilde{\mathbf{M}}$ contains the matrix $\mathbf{M}$ with rows and columns replaced by unit crosses for "out" variables: [27]

$$\mathbf{M} = \begin{bmatrix} \mathbf{C} & \mathbf{A}' \\ \mathbf{A} & \mathbf{0} \end{bmatrix}, \tag{8}$$

A clarification regarding "out" variables is in order. According to Sharpe (1970) three states exists for variables involved in the critical line algorithm solution: "out" variables, which are those that take on their lower bound value; "in" variables that refer to those whose values are located between bounds; and "up" variables to which upper bound values are assigned. For example, if we were to optimize a portfolio of three assets with infinite risk tolerance and subject to only one constraint (the sum of assets must be one), the problem would be solved by investing all in the asset with the highest return. Thus, "out" variables would be the remaining assets, since they would be assigned their lower bound value (zero).

[26] See Appendix B to understand the corner portfolio concept with the help of a graphical example taken from Markowitz (1959).

[27] When a variable is out, original values are replaced by ones in every row and column that correspond to that variable. According to Markowitz (1959), it is more convenient to write zero crosses than unit crosses for computational purposes.

The matrix $\mathbf{R}$ is given by:

$$\mathbf{R} = \begin{bmatrix} \mathbf{0} \\ \mathbf{b} \end{bmatrix}, \tag{9}$$

while the $\widetilde{\mathbf{S}}$ matrix is formed by the column-vector $\mathbf{S}$ with zeros corresponding to "out" variables:

$$\mathbf{S} = \begin{bmatrix} \boldsymbol{\mu} \\ \mathbf{0} \end{bmatrix}, \tag{10}$$

In the critical line formula established previously, λ_E, represents the Lagrange multiplier associated with the constraint that funds must be fully invested $\left(\sum X_j = 1\right)$, while there exists one lambda $\left(\boldsymbol{\lambda} = \left[\lambda_1, \ldots \lambda_\mu\right]\right)$ for each constraint equation.

Therefore, the formula to obtain the critical line may be written as:

$$\begin{bmatrix} \mathbf{X} \\ \boldsymbol{\lambda} \end{bmatrix} = \left(\widetilde{\mathbf{M}}\right)^{-1}\mathbf{R} + \left(\widetilde{\mathbf{M}}\right)^{-1}\widetilde{\mathbf{S}}\boldsymbol{\lambda}_{\mathbf{E}}\,. \tag{11}$$

After some computations, we can obtain different critical lines, which will form the set of efficient portfolios.

8. Application of the General Problem to Mexican Pension Funds

In order to obtain the mean-variance efficient portfolio for Mexican pension funds, we apply the Markowitz optimization algorithm previously described. More specifically, we use 70 indices (69 stock indices and 1 fixed income index) and solve the problem subject to either the new or the old CONSAR's (*Comisión Nacional del Sistema de Ahorro para el Retiro*) investment regime.

For this particular problem, weights can be denoted as:

$$\mathbf{X_j} = \left[\mathbf{X_f}, \mathbf{X_d}, \mathbf{X_{rf}}\right]', \tag{12}$$

where:

$\mathbf{X_f}$ is a vector of weights for foreign indices,

$\mathbf{X_d}$ is a vector of weights for domestic indices,

$\mathbf{X_{rf}}$ is the weight for the default risk-free index.

In this case, the first 67 weights correspond to foreign indices (widely followed market and MSCI indices), the next two indices correspond to domestic indices (IPC and INMEX) while the last weight corresponds to the default risk-free index.

When considering the new CONSAR's investment regime, the vector $\mathbf{X_j}$ is subject to the following constraints:[28]

$$\sum X_d \leq 0.30;\ \sum X_f \leq 0.20 \ \ and \ \ \sum X_f + \sum X_d \leq 0.30 \text{ if worker's age is <27} \tag{13}$$

$$\sum X_d \leq 0.25;\ \sum X_f \leq 0.20 \ \ and \ \ \sum X_f + \sum X_d \leq 0.25 \text{ if } 27 \leq \text{worker's age<37} \tag{14}$$

$$\sum X_d \leq 0.20;\ \sum X_f \leq 0.20 \ \ and \ \ \sum X_f + \sum X_d \leq 0.20 \text{ if } 37 \leq \text{worker's age<46} \tag{15}$$

$$\sum X_d \leq 0.15;\ \sum X_f \leq 0.15 \ \ and \ \ \sum X_f + \sum X_d \leq 0.15 \text{ if } 46 \leq \text{worker's age<56} \tag{16}$$

$$\sum X_d = 0;\ \sum X_f = 0 \ \ and \ \ \sum X_f + \sum X_d = 0 \quad \text{if worker's age} \geq 56 \tag{17}$$

$$\sum X_j = 1 \tag{18}$$

$$X_j \geq 0 \tag{19}$$

[28] *Circular CONSAR 15-8, Circular CONSAR 15-19 and Circular CONSAR 15-20.*

It is important to note that this set of constraints represents the aforementioned expression $\mathbf{AX_j} = \mathbf{b}$.

In contrast, the old CONSAR's investment regime implies that the vector $\mathbf{X_j}$ is subject to constraints (18), (19) and the following:[29]

$$\sum X_d \leq 0.15;\ \sum X_f \leq 0.15 \ \ and \ \ \sum X_f + \sum X_d \leq 0.15 \ \text{if worker's age<56} \qquad (20)$$

$$\sum X_d = 0;\ \sum X_f = 0 \ \ and \ \ \sum X_f + \sum X_d = 0 \qquad \text{if worker's age} \geq 56 \qquad (21)$$

Thus, the particular problem takes the form expressed in the previous section and is solved using the critical line algorithm. To obtain the mean-variance efficient frontier we use the portfolio optimization function from the MATLAB's Financial Toolbox. See Appendix C for further information regarding this function.

9. Simulation Results

In this section we present the main findings from the simulation analysis. In particular, we are interested in determining if the new and old investment regimes have different impacts on the probability of exercising the Minimum Pension Guarantee (MPG). Furthermore, we analyze the role of the level of commissions by comparing their effects on such probability under the new investment rules. The analysis is done for both constant wage levels and the more realistic assumption of earnings being determined by a lifetime wage profile of the average worker. See Appendix D for the derivation of such lifetime wage profile.

To determine if the new investment regime represents an improvement over the old investment rules, we do a significance test for the difference between the probabilities of exercising the Minimum Pension Guarantee (MPG) under both regimes. We conduct this experiment for 1, 2, 3, 4, 5, 10, 15 and 25 minimum wages to find if there might be a differentiated outcome when contrasting the probabilities. Moreover, by running the same experiment under the assumption of a lifetime wage profile, it allows us to compare the new investment regime effects on such probability to the old one's in a more realistic environment.

[29] *Circular CONSAR 15-12.*

The rest of the analysis focuses on finding the effect of the levels of commissions on the probability of exercising the MPG. This is done to understand the extent to which lowering commissions affects such probability. In doing this experiment, we consider five levels of commissions charged on balances once a year: the most expensive, the second most expensive, the cheapest, the second cheapest and the system's average.

The experiments can be summarized in the following six cases:

Case I: Obtaining the Probability of exercising the Minimum Pension Guarantee for different wage levels given average commissions and the new investment regime.

Case II: Obtaining the Probability of exercising the Minimum Pension Guarantee for different wage levels given average commissions and the old investment regime.

Case III: Obtaining the Probability of exercising the Minimum Pension Guarantee for a lifetime wage profile given average commissions and the new investment regime.

Case IV: Obtaining the Probability of exercising the Minimum Pension Guarantee for a lifetime wage profile given average commissions and the old investment regime.

Case V: Obtaining the Probability of exercising the Minimum Pension Guarantee given different levels of both commissions and wages under the new investment regime.

Case VI: Obtaining the Probability of exercising the Minimum Pension Guarantee given a lifetime wage profile and different levels of commissions under the new investment regime.

9.1 The probability of exercising the Minimum Pension Guarantee: the new vs. the old investment regime

Under the new investment regime, workers earning between one and two minimum wages have a statistically significantly lower probability of exercising the Minimum Pension Guarantee (MPG) along most of the efficient frontier. For the other wage levels analyzed in this study, allowing higher maximum limits for investing in variable income does not result in a statistical reduction of the probability of exercising the MPG.

Case I: Obtaining the Probability of exercising the Minimum Pension Guarantee for different wage levels given average commissions and the new investment regime.

Table I shows the probability of exercising the MPG for various wage levels given average commissions and the new investment regime. It can be seen that such probability decreases as the wage level increases and as we move from the minimum variance portfolio (Port1) to the maximum return portfolio (Port10) along the efficient frontier.

Table I. Probability of exercising the Minimum Pension Guarantee for different wage levels given average commissions and the new investment regime.

25 years of contribution

Number of Minimum Wages	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10
1	0.998	0.991	0.973	0.913	0.822	0.719	0.612	0.522	0.438	0.373
2	0.818	0.661	0.479	0.342	0.254	0.177	0.141	0.117	0.095	0.086
3	0.169	0.076	0.040	0.023	0.014	0.013	0.013	0.012	0.013	0.013
4	0.006	0.004	0.003	0.003	0.003	0.003	0.003	0.002	0.001	0.002
5	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Case II: Obtaining the Probability of exercising the Minimum Pension Guarantee for different wage levels given average commissions and the old investment regime.

We also computed the probabilities of resorting to the MPG under the old regime rules due to our interest in determining if the new investment regime has a different impact on such probabilities.

Table II shows the probability of exercising the MPG for various wage levels given average commissions and the old investment regime. Just like in Table I, it can be seen that such probability also decreases as the wage level increases and as we move from the minimum variance portfolio (Port1) to the maximum return portfolio (Port10) along the efficient frontier.

Table II. Probability of exercising the Minimum Pension Guarantee for different wage levels given average commissions and the old investment regime.

25 years of contribution										
Number of Minimum Wages	**Port1**	**Port2**	**Port3**	**Port4**	**Port5**	**Port6**	**Port7**	**Port8**	**Port9**	**Port10**
1	0.997	0.993	0.986	0.965	0.920	0.859	0.777	0.704	0.620	0.556
2	0.819	0.709	0.588	0.468	0.351	0.279	0.219	0.178	0.147	0.130
3	0.160	0.089	0.056	0.033	0.019	0.015	0.013	0.012	0.011	0.011
4	0.006	0.003	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

To determine if the new investment regime results in a statistical reduction in the probability of resorting to the MPG, we do a statistical test at the 5% level of significance for the difference between the probabilities from Tables I and II. In following the test proposed by Fernández and Díaz (2001), we find that only workers earning between one and two minimum wages have a statistically significantly lower probability of exercising the Minimum Pension Guarantee (MPG) along most of the efficient frontier. See Appendix E for details on how to do the hypothesis testing and Tables E.2-E.6 for test results.

Consequently, the new investment regime represents an improvement over the old one for the two lowest wage groups. This finding is important for a couple of reasons. First and foremost, 76.2% of Mexican workers belong to these wage groups.[30] Second, as it can be seen from Table I, these particular groups have the highest probabilities of resorting to the MPG.

Case III: Obtaining the Probability of exercising the Minimum Pension Guarantee for a lifetime wage profile given average commissions and the new investment regime.

Since we think that the assumption of keeping the number of minimum wages constant is somewhat unrealistic, we use data from the Mexican labor market to determine if a lifetime wage profile of an average worker could reduce the probability of exercising the MPG. Table III shows the probability of exercising the MPG given average commissions and a lifetime wage profile under the new investment regime.

Table III. Probability of exercising the Minimum Pension Guarantee for a lifetime wage profile given average commissions and the new investment regime.

[30] See Roldán-Flores, Domínguez-Paredes and Madero-Suárez (2006) for more details on the distribution of workers according to their wage level.

Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10
0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

From the table above, it can be seen that such probability is very close to zero or nil along most of the efficient frontier. By comparing the probabilities from Table III to those from Table I, it can be noticed that they are much lower than those corresponding to three minimum wages or lower wage levels. Therefore, the lifetime wage profile of an average worker seems to indicate that the assumption of relatively low constant real wages overestimates the probability of exercising the MPG.

Case IV: Obtaining the Probability of exercising the Minimum Pension Guarantee for a lifetime wage profile given average commissions and the old investment regime.

Running the same experiment under the assumption of a lifetime wage profile will allow us to compare the new investment regime effects on such probability to the old one's in a more realistic environment. Table IV shows the probability of exercising the MPG given average commissions and a lifetime wage profile under the old investment regime.

Table IV. Probability of exercising the Minimum Pension Guarantee for a lifetime wage profile given average commissions and the old investment regime.

Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10
0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Just like in Table III, it can be seen that such probability is very close to zero or nil along the efficient frontier. By comparing the probabilities from the table above to those from Table II, it can be noticed that they are much lower than those corresponding to three minimum wages or lower wage levels. Once again, these results seem to provide some evidence that the assumption of relatively low constant real wages overestimates the probability of exercising the MPG.

9.2 The effect of the levels of commissions on the probability of exercising the Minimum Pension Guarantee under the new investment regime

Under the new investment regime, we find that there is statistical difference between the probabilities corresponding to the groups of the cheapest and most expensive commissions at the 5% level of significance and for one minimum wage only, but not for the difference between the probabilities corresponding to both groups for other wage levels.

Case V: Obtaining the Probability of exercising the Minimum Pension Guarantee given different levels of both commissions and wages under the new investment regime.

Relative expensive commissions could be one of the reasons for explaining high probabilities of exercising the MPG. Finding the sensitivity of such probability to commissions is done to understand the extent to which lowering these charges reduces the likelihood of resorting to the guarantee. For this experiment, we consider five levels of commissions charged on balances once a year: the most expensive, the second most expensive, the cheapest, the second cheapest and the system's average. Table V shows the probability of exercising the MPG for different combinations of wages and commissions given Portfolio 10 and the new investment regime.[31]

Table V. Probability of exercising the Minimum Pension Guarantee given Portfolio 10 and different levels of both commissions and wages.

25 years of contribution

Number of Minimum Wages	Most expensive Commission		Average Commission	Cheapest Commission	
	First	Second		Second	First
1	0.411	0.396	0.373	0.353	0.351
2	0.109	0.098	0.086	0.080	0.080
3	0.016	0.015	0.013	0.010	0.010
4	0.003	0.002	0.002	0.001	0.001
5	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000

Ceteris paribus, it can be seen from Table V that the probability of resorting to the guarantee decreases as commissions become cheaper. However, at the 5% level of significance and for one minimum wage only, there is statistical difference between the probabilities corresponding to the groups of the cheapest and most expensive commissions, but not for the difference between the probabilities corresponding to both groups for the other wage levels. See Table E.7 in Appendix E for test results.

[31] Since portfolio 10 brings about the lowest probabilities of exercising the MPG when compared to other portfolios, we would like to find out if reductions in commissions could translate into even statistically significantly lower probabilities.

Case VI: Obtaining the Probability of exercising the Minimum Pension Guarantee given a lifetime wage profile and different levels of commissions under the new investment regime.

It is interesting to find the effect of commissions levels on the probability of resorting to the MPG when a lifetime wage profile is assumed. Table VI shows the probability of exercising the MPG for different levels of commissions given a lifetime wage profile and the new investment regime.

Table VI. Probability of exercising the Minimum Pension Guarantee given a lifetime wage profile and different levels of commissions.

Portfolio Number	Most expensive Commission		Average Commission	Cheapest Commission	
	First	Second		Second	First
Port1	0.004	0.003	0.001	0.001	0.001
Port2	0.001	0.001	0.001	0.000	0.000
Port3	0.000	0.000	0.000	0.000	0.000
Port4	0.000	0.000	0.000	0.000	0.000
Port5	0.000	0.000	0.000	0.000	0.000
Port6	0.000	0.000	0.000	0.000	0.000
Port7	0.000	0.000	0.000	0.000	0.000
Port8	0.000	0.000	0.000	0.000	0.000
Port9	0.000	0.000	0.000	0.000	0.000
Port10	0.000	0.000	0.000	0.000	0.000

It can be seen from Table VI that the probability of resorting to the guarantee either slightly decreases or does not change at all as commissions become cheaper. At the 5% level of significance, there is no statistical difference between the probabilities corresponding to the groups of the cheapest and most expensive commissions. See Table E.8 in Appendix E for test results.

It is important to mention that one of the recently approved regulatory changes consists of allowing AFORES to only charge commissions on balances, not on inflows anymore.[32] Furthermore, new workers will be assigned to AFORES with the highest net return track record, and no longer to funds with the lowest commissions. Therefore, pension fund managers will

[32] For a thirteen-country sample, Whitehouse (2001) finds that commissions on inflows or the *charge ratio* in the Mexican pension fund system is the second most expensive after Australia's.

have the incentive to obtain higher net returns in order to grow balances faster. The analysis done in this work assumes equivalent commissions on balances, implying that commissions on inflows are also included in these numbers. It is possible that the new competition rules could push commissions on balances lower, thus making the need to resort to the MPG less likely.

10. Conclusions

Important modifications were introduced to the investment regime to increase portfolio diversification and possibly to obtain higher returns by increasing the limits to investment in variable income securities. If higher returns could be obtained, this would have effects on final account balances and, therefore, on the probability of exercising the minimum pension guarantee.

By a systematic process of portfolio optimization in which the efficient frontier is obtained, our results show that AFORES could take advantage of the new investment regime to increase final balances of retirement accounts. This finding is particularly relevant to the probability of exercising the MPG for low-income workers (earners of one and two minimum wages). Hypothesis testing shows that such probability is statistically significantly lower (with a 95% confidence interval) than that corresponding to the old investment regime. Moreover, as we move from the minimum-variance portfolio to the maximum-return portfolio along the efficient frontier the probability of exercising the MPG considerably decreases for low income workers.

When we analyze workers earning at least four minimum wages, the probability of exercising the MPG is either close to zero or nil regardless of the point on the efficient frontier. Similarly, when we consider the life-cycle salary profile of the average worker, the probability that this worker resorts to the minimum pension guarantee is either zero or almost nil. Finally, differences in the commission levels charged by the cheapest and most expensive groups of AFORES only have a statistically significant effect on those workers earning just one minimum wage.

It is important to mention that our results rely on a month-to-month process of portfolio optimization by pension fund administrators. Such process assumes a multivariate normal distribution of assets returns that will not change in the future. However, we hope that this work highlights the importance of engaging in active and efficient portfolio management to make the most of the new investment regime.

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Appendix A

This appendix shows a list with the 70 stock indices used in the construction of optimal portfolios.

1	Germany	DAX
2	Australia	ASX 50
3	Belgium	BEL20
4	Canada	TSX
5	Spain	IBEX-35
6		IGBM
7	United States	AMEX Composite
8		Dow Jones Industrial Average
9		Dow Jones Composite Average
10		Dow Jones Global Titans 50
11		Dow Jones Global Titans 50 Euro
12		Dow Jones Stoxx Global 1800
13		Dow Jones Stoxx 50
14		Dow Jones Euro Stoxx 50
15		Nasdaq Composite
16		NYSE Composite
17		NYSE International 100
18		Standard and Poor's Global 100
19		Standard and Poor's Global 1200
20		Standard and Poor's 100
21		Standard and Poor's 500
22		Standard and Poor's 400 MidCap
23		Standard and Poor's 600 Small Cap
24		Standard and Poor's 1500 Supercomposite
25		Standard and Poor's Europa 350
26		Russell 3000
27		Dow Jones Wilshire 5000 Composite
28		Dow Jones US Total Market Index (TMI).
29	France	CAC 40
30	Holland	AEX
31	Hong Kong	HANG SENG
32	United Kingdom	FTSE 350
33		FTSE 250
34		FTSE 100
35	Italy	MIBTEL
36		MIB30
37	Japan	NIKKEI
38		TOPIX
39	Switzerland	SMI
40		SPI
41	Mexico	IPC
42		INMEX
43	MSCI	Europe
44		Pan Europe
45		Euro
46		EMU
47		Pacific
48		Far East
49		North America

50		EAFE
51		EASEA
52		World
53		Kokusai
54		UK
55		Japan
56		US Investable Market 2500
57		US Prime Market 750
58		US Small Cap 1750
59		US Mid Cap 450
60		US Large Cap 300
61		Germany
62		Austria
63		Australia
64		Canada
65		France
66		Holland
67		Hong Kong
68		Italy
69		Sweden
70	Mexico	CETES 28 days

Appendix B

This appendix has the purpose of illustrating the concept of corner portfolio with an example involving three securities. In Figure 1, the horizontal axis represents the amount invested in the first security, while the vertical axis represents the amount invested in the second security. Then the point *A* denotes a portfolio with 25% invested in the first security and 50% in the second security. Since the sum of the security weights is equal to one, the investment in the third security must be 25%.

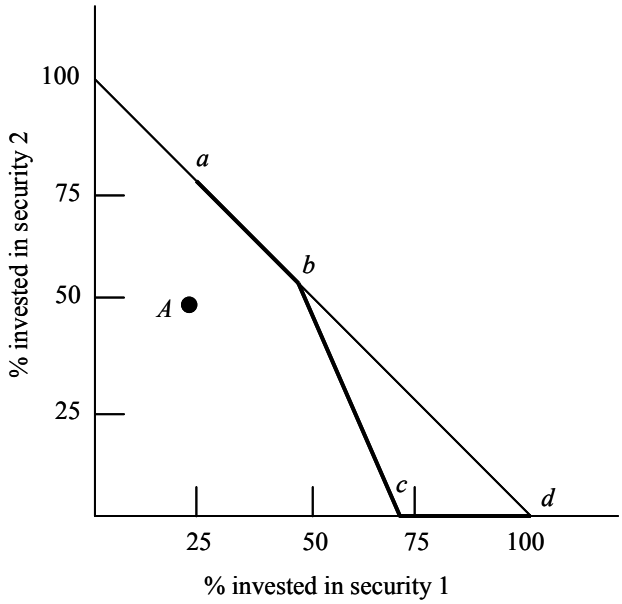


Figure B.1. Efficient portfolios formed with three securities.

The dark line of Figure 1 shows the set of efficient portfolios formed with three securities. Every point on the dark line is efficient. The locus of points representing efficient portfolios starts with point *a*, which has the smallest return variance. The locus moves from point *a* to *b*, there it kinks and moves on another straight line until it reaches *c*, then, it kinks again and moves until reaching point *d*, which has the largest return variance.

If points *a*, *b*, *c* and *d* are known, other points representing efficient portfolios can be inferred. This set of points (*a*, *b*, *c* and *d*) represents the corner portfolios of this example. It is important to note that each corner portfolio is efficient, as well as any linear combinations of consecutive corner portfolios.

From the example above, we can note that portfolio *b* contains one more security than both portfolio *a* and portfolio *c.* The latter also has one more security than portfolio *d*. Then, the number of securities in efficient portfolios is either increased or decreased by one when a consecutive corner portfolio arises.

Appendix C

In this appendix we provide a brief description of the *portopt* function contained in the Financial Toolbox of MATLAB.

To obtain the mean-variance efficient portfolios, we use the *portopt* (portfolio optimization) function from MATLAB's Financial Toolbox. The function produces the mean-variance efficient frontier described by Markowitz (1959). The function requires a covariance matrix $\mathbf{C}$, a vector of expected returns $\boldsymbol{\mu}_j$ and a constraint matrix $\mathbf{AX}_j = \mathbf{b}$ as inputs.

In our case, the constraint matrix $\mathbf{AX}_j = \mathbf{b}$ complies with the CONSAR's new or old investment regime regulations. For example, under the new regime the constraint matrix is constructed by equations (13)-(19) from Section 8 in the text.

One of the advantages of the *portopt* function is that we can specify the number of portfolios along the efficient frontier. For each simulation we generate ten efficient portfolios with their corresponding optimal assets weights, expected return and variance. Such set includes the minimum variance and the maximum return portfolios.

The *portopt* function could also be used to plot the efficient frontier. The minimum risk portfolio is represented by the closest point to the vertical axis while the maximum return portfolio is shown by the most distant point to the same axis.

Appendix D

In this appendix we describe the procedure to derive a lifetime wage distribution of the average worker.

Ideally, we would like to have official data for the distribution of average wages across age groups for workers of the formal private sector affiliated to the Mexican Institute of Social Security (*Instituto Mexicano del Seguro Social, IMSS*). Unfortunately, there are not such data. As a consequence we must construct the aforementioned wage distribution based on available data.

In order to construct a lifetime salary profile for the average worker, we used data from two main sources: the National Survey of Employment and Occupation (*Encuesta Nacional de Ocupación y Empleo, ENOE*) published by INEGI and the National Commission of Minimum Wages (*Comisión Nacional de Salarios Mínimos, CONASAMI*).

The ENOE contains quarterly data on hourly wages and the number of hours worked per week for different age-groups. Based on these data we obtained (by simple multiplication) the weekly wage for each age-group. We proceeded to divide the weekly wage by seven to obtain a daily wage.

We decided against using this daily wage as a proxy for the average daily salary of workers affiliated to the AFORES pension system due to the following:

i. ENOE's data are representative for the whole occupied population (formal and informal) from the private and public sectors. Since the AFORES pension system of individual accounts only applies to workers of the formal private sector this proxy would be biased.
ii. Contribution rates to the pension system of individual accounts are determined based on the quoted salary of the worker (*salario de cotización*) which typically excludes non-salary benefits. However, hourly wages published by ENOE includes non-salary benefits and is net of taxes and contributions to unions and social security systems.
iii. Hourly wages published by ENOE may combine two or more sources of income from different employment positions for a single worker. However, the contributions paid by workers affiliated to IMSS for their retirement accounts are based on a single employment position salary.

Once we had taken into account the previous considerations, we derived the weight for each group using ENOE data. The following tables show our calculations:

Table D.1. Daily income for the whole occupied population and weights.

Daily Income Derived from Hours Worked per Week (Nominal Pesos)											
	I-2005	II-2005	III-2005		I-2006	II-2006	III-2006	IV-2006	I-2007	II-2007	III-2007
All cohorts	*150.55*	*149.87*	*152.90*	*156.14*	*159.09*	*161.41*	*163.99*	*166.37*	*169.74*	*172.71*	*171.49*
14 a 19 years old	76.19	77.34	79.17	79.48	80.79	83.41	88.85	86.16	87.73	89.98	91.32
20 a 24 years old	116.55	115.00	119.06	124.32	123.58	128.60	131.03	132.05	132.29	135.68	142.86
25 a 29 years old	147.72	150.09	152.72	154.56	158.27	160.43	162.13	166.23	180.51	171.67	172.86
30 a 34 years old	163.97	166.35	167.67	168.29	174.69	179.26	179.09	181.89	182.33	185.38	184.31
35 a 39 years old	168.18	174.66	177.93	179.50	183.30	183.94	183.85	188.87	190.31	197.69	194.12
40 a 44 years old	181.88	181.49	182.89	188.76	189.64	192.82	195.00	193.93	199.87	207.16	200.97
45 a 49 years old	189.27	185.14	188.40	191.62	192.47	196.26	207.27	202.86	208.50	215.27	209.41
50 a 54 years old	168.76	172.64	177.90	182.18	191.56	196.23	189.88	203.17	202.43	197.12	197.88
55 a 59 years old	155.13	153.23	157.15	162.09	170.72	162.15	177.46	175.94	170.43	196.38	172.30
60 a 64 years old	141.28	131.26	144.28	149.09	135.73	141.62	143.43	148.34	149.81	149.40	159.12
65 years and older	140.65	100.62	111.27	113.73	112.60	113.20	112.74	116.51	120.81	114.61	131.16

Source: Our calculations with ENOE data.

Weights											
	I-2005	II-2005	III-2005	IV-2005	I-2006	II-2006	III-2006	IV-2006	I-2007	II-2007	III-2007
All cohorts	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*
14 a 19 years old	0.51	0.52	0.52	0.51	0.51	0.52	0.54	0.52	0.52	0.52	0.53
20 a 24 years old	0.77	0.77	0.78	0.80	0.78	0.80	0.80	0.79	0.78	0.79	0.83
25 a 29 years old	0.98	1.00	1.00	0.99	0.99	0.99	0.99	1.00	1.06	0.99	1.01
30 a 34 years old	1.09	1.11	1.10	1.08	1.10	1.11	1.09	1.09	1.07	1.07	1.07
35 a 39 years old	1.12	1.17	1.16	1.15	1.15	1.14	1.12	1.14	1.12	1.14	1.13
40 a 44 years old	1.21	1.21	1.20	1.21	1.19	1.19	1.19	1.17	1.18	1.20	1.17
45 a 49 years old	1.26	1.24	1.23	1.23	1.21	1.22	1.26	1.22	1.23	1.25	1.22
50 a 54 years old	1.12	1.15	1.16	1.17	1.20	1.22	1.16	1.22	1.19	1.14	1.15
55 a 59 years old	1.03	1.02	1.03	1.04	1.07	1.00	1.08	1.06	1.00	1.14	1.00
60 a 64 years old	0.94	0.88	0.94	0.95	0.85	0.88	0.87	0.89	0.88	0.87	0.93
	0.93		0.73		0.71		0.69	0.70	0.71	0.66	0.76

Source: Our calculations with ENOE data.

Since the CONASAMI only publishes the average daily salary for all workers affiliated to IMSS (see Table D.2), we used the weights derived above to calculate the average daily salary for all age groups of workers affiliated to IMSS.

Table D.2. Quoted average daily salary of affiliated workers to IMSS.

Quoted Average Daily Salary of IMSS-Affiliated Workers (nominal pesos)											
	I-2005	II-2005	III-2005	IV-2005	I-2006	II-2006	III-2006	IV-2006	I-2007	II-2007	III-2007
CONASAMI	190.76	188.62	190.14	189.34	196.84	198.06	199.73	199.29	208.44	209.11	209.95

Source: CONASAMI

By multiplying the weights from Table D.1 by the national quoted average daily salary of IMSS-affiliated workers from Table D.2, we obtain a "distribution" of this salary by age-group. The results are shown in Table D.3.

Table D.3. Derived quoted average daily salary of affiliated workers to IMSS by age group.

Quoted Average Daily Salary of IMSS-Affiliated Workers (nominal pesos)											
	I-2005	II-2005	III-2005	IV-2005	I-2006	II-2006	III-2006	IV-2006	I-2007	II-2007	III-2007
Total	*190.76*	*188.62*	*190.14*	*189.34*	*196.84*	*198.06*	*199.73*	*199.29*	*208.44*	*209.11*	*209.95*
14 a 19 years old	96.54	97.33	98.45	96.38	99.96	102.35	108.21	103.21	107.73	108.95	111.81
20 a 24 years old	147.67	144.74	148.06	150.76	152.90	157.79	159.60	158.19	162.45	164.27	174.90
25 a 29 years old	187.17	188.91	189.91	187.43	195.83	196.85	197.47	199.13	221.67	207.85	211.64
30 a 34 years old	207.76	209.37	208.51	204.07	216.15	219.95	218.13	217.88	223.90	224.45	225.66
35 a 39 years old	213.09	219.83	221.26	217.67	226.79	225.70	223.92	226.25	233.69	239.35	237.66
40 a 44 years old	230.46	228.42	227.44	228.90	234.63	236.59	237.50	232.31	245.44	250.82	246.04
45 a 49 years old	239.82	233.01	234.29	232.36	238.14	240.81	252.45	243.01	256.03	260.64	256.38
50 a 54 years old	213.83	217.29	221.22	220.92	237.01	240.77	231.26	243.38	248.58	238.66	242.26
55 a 59 years old	196.57	192.86	195.42	196.56	211.23	198.96	216.14	210.76	209.29	237.77	210.95
60 a 64 years old	179.01	165.21	179.42	180.79	167.93	173.77	174.70	177.70	183.97	180.88	194.80
65 years and older	178.22	126.64	138.37	137.91	139.32	138.90	137.31	139.57	148.36	138.76	160.58

Source: Own calculations with ENOE and CONASAMI data.

In order to express this average daily salary in numbers of minimum wages, we used data available from Banco de México for the general minimum wage at the end of each quarter. This information is shown in Table D.4.

Table D.4. Minimum wage per day.

Daily Minimum Wage (nominal pesos)											
	I-2005	II-2005	III-2005	IV-2005	I-2006	II-2006	III-2006	IV-2006	I-2007	II-2007	III-2007
General Minimum Wage	*45.24*	*45.24*	*45.24*	*45.24*	*47.05*	*47.05*	*47.05*	*47.05*	*48.88*	*48.88*	*48.88*

Source: Banco de México.

Finally, we divided the data from Table D.3 by the minimum wages from Table D.4 to obtain the derived quoted average daily salary of workers affiliated to IMSS by age group as a multiple of the minimum wage. Our results are shown in Table D.5.

Table D.5. Multiple of the minimum wage for workers affiliated to IMSS by age group.

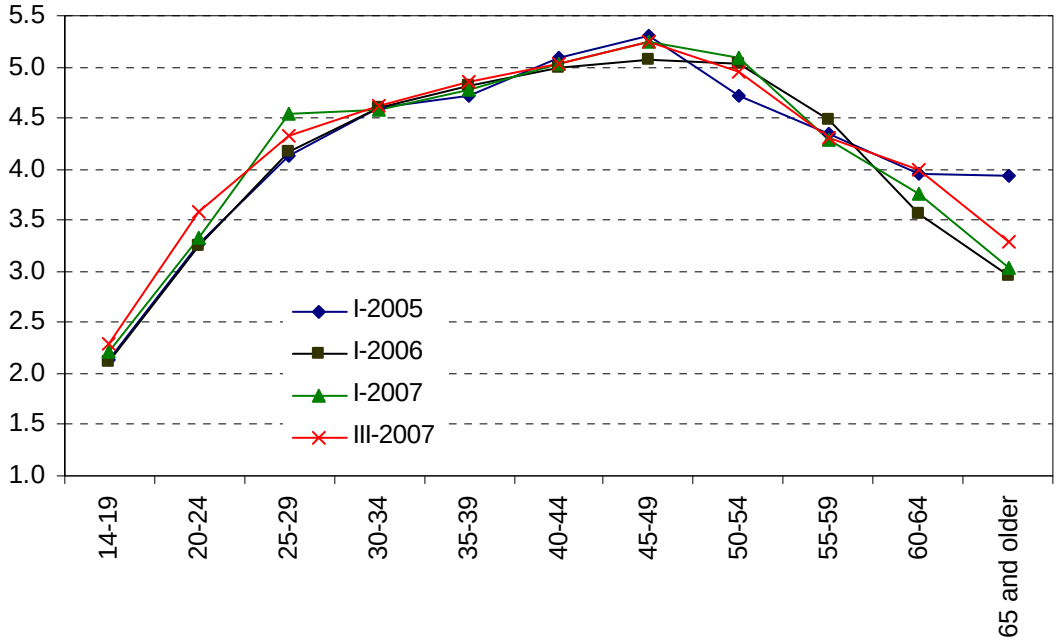
Quoted Average Daily Salary of IMSS-Affiliated Workers (multiples of the general minimum wage)											
	I-2005	II-2005	III-2005	IV-2005	I-2006	II-2006	III-2006	IV-2006	I-2007	II-2007	III-2007
Total	*4.22*	*4.17*	*4.20*	*4.19*	*4.18*	*4.21*	*4.25*	*4.24*	*4.26*	*4.28*	*4.30*
14 a 19 years old	2.13	2.15	2.18	2.13	2.12	2.18	2.30	2.19	2.20	2.23	2.29
20 a 24 years old	3.26	3.20	3.27	3.33	3.25	3.35	3.39	3.36	3.32	3.36	3.58
25 a 29 years old	4.14	4.18	4.20	4.14	4.16	4.18	4.20	4.23	4.53	4.25	4.33
30 a 34 years old	4.59	4.63	4.61	4.51	4.59	4.67	4.64	4.63	4.58	4.59	4.62
35 a 39 years old	4.71	4.86	4.89	4.81	4.82	4.80	4.76	4.81	4.78	4.90	4.86
40 a 44 years old	5.09	5.05	5.03	5.06	4.99	5.03	5.05	4.94	5.02	5.13	5.03
45 a 49 years old	5.30	5.15	5.18	5.14	5.06	5.12	5.37	5.16	5.24	5.33	5.25
50 a 54 years old	4.73	4.80	4.89	4.88	5.04	5.12	4.92	5.17	5.09	4.88	4.96
55 a 59 years old	4.35	4.26	4.32	4.34	4.49	4.23	4.59	4.48	4.28	4.86	4.32
60 a 64 years old	3.96	3.65	3.97	4.00	3.57	3.69	3.71	3.78	3.76	3.70	3.99
65 years and older	3.94	2.80	3.06	3.05	2.96	2.95	2.92	2.97	3.04	2.84	3.29

Source: Own calculations using data from Banco de México, CONASAMI and ENOE.

The following graph shows some of the results from Table D.5.

Figure D.1. Life-cycle salary profile for the average worker.

(in number of minimum wages)



Appendix E

In this appendix we explain the procedure used to determine if there is statistical difference between two probabilities of exercising the Minimum Pension Guarantee (MPG). To explain the procedure, we rely on an example which contains some of the results obtained in this work.

We have two investment regimes: the New Regime (A) and the Old Regime (B). Their goal is to increase balances of retirement accounts above a minimum level, that is, the Minimum Pension Guarantee (MPG). We repeat the balances accumulation experiment one thousand times under both regimes. For a worker earning two minimum wages and investing her balances in Port9, the simulation results indicate that 95 and 147 cases obtain lower balances than the MPG under the New Regime and Old Regime, respectively. Is there statistical difference between both regimes?

To answer this question, we need to establish two hypotheses:

H_0 (null hypothesis) = There is no statistical difference between the two regimes;
H_a (alternative hypothesis) = There is statistical difference between the two regimes.

Additionally, the probabilities obtained for each regime are:

Table E.1. Probabilities of exercising the MPG under the new and old regimes.

Investment Regime	Repetitions (*N*)	Probabilities
New Regime	1,000	95/1,000 = 0.095
Old Regime	1,000	147/1,000 = 0.147

By taking into account the information from the table above and assuming a 5% level of significance, we proceed to calculate the following:

$$\left|p_A - p_B\right| = \left|0.095 - 0.147\right| = 0.052 \tag{E.1}$$

$$p = \frac{p_A + p_B}{2} = \frac{0.095 + 0.147}{2} = 0.121 \tag{E.2}$$

$$z_{\alpha=0.05} = 1.96 \tag{E.3}$$

$$\text{Standard Error (S.E.)} = \sqrt{p(1-p)\left(1/N + 1/N\right)}$$
$$= \sqrt{0.121(1-0.121)\left(1/1{,}000 + 1/1{,}000\right)} \quad \text{(E.4)}$$
$$= 0.015$$

$$\text{S.E.} * 1.96 = 0.015 * 1.96 = 0.029 \quad \text{(E.5)}$$

If $|p_A - p_B|$ is greater than the standard error times 1.96, we conclude that this difference is statistically significant. In this case $0.052 > 0.029$, then the difference between 0.095 and 0.147 is statistically significant. Therefore, we can reject the null hypothesis at a 5% level of significance.

The following tables contain the calculations needed to determine if there is statistical difference between two probabilities of exercising the MPG.

Table E.2. Hypothesis testing to determine if there is statistical difference between the probabilities of exercising the MPG under the new and the old investment regimes for workers earning one minimum wage.

One Minimum Wage	Probability of Exercising the MPG		Abs(NR-OR)	(NR+OR)/2	Standard Error	Standard Error*1.96
	New Regime (NR)	Old Regime (OR)				
Port1	0.998	0.997	0.001	0.998	0.002	0.004
Port2	0.991	0.993	0.002	0.992	0.004	0.008
Port3*	0.973	0.986	0.013	0.980	0.006	0.012
Port4*	0.913	0.965	0.052	0.939	0.011	0.021
Port5*	0.822	0.920	0.098	0.871	0.015	0.029
Port6*	0.719	0.859	0.140	0.789	0.018	0.036
Port7*	0.612	0.777	0.165	0.695	0.021	0.040
Port8*	0.522	0.704	0.182	0.613	0.022	0.043
Port9*	0.438	0.620	0.182	0.529	0.022	0.044
Port10*	0.373	0.556	0.183	0.465	0.022	0.044

*The symbol * indicates that there is statistical difference at the 5% level of significance.*

Table E.3. Hypothesis testing to determine if there is statistical difference between the probabilities of exercising the MPG under the new and the old investment regimes for workers earning two minimum wages.

Two Minimum Wages	Probability of Exercising the MPG		Abs(NR-OR)	(NR+OR)/2	Standard Error	Standard Error*1.96
	New Regime (NR)	Old Regime (OR)				
Port1	0.818	0.819	0.001	0.819	0.017	0.034
Port2*	0.661	0.709	0.048	0.685	0.021	0.041
Port3*	0.479	0.588	0.109	0.534	0.022	0.044
Port4*	0.342	0.468	0.126	0.405	0.022	0.043
Port5*	0.254	0.351	0.097	0.303	0.021	0.040
Port6*	0.177	0.279	0.102	0.228	0.019	0.037
Port7*	0.141	0.219	0.078	0.180	0.017	0.034
Port8*	0.117	0.178	0.061	0.148	0.016	0.031
Port9*	0.095	0.147	0.052	0.121	0.015	0.029
Port10*	0.086	0.130	0.044	0.108	0.014	0.027

*The symbol * indicates that there is statistical difference at the 5% level of significance.*

Table E.4. Hypothesis testing to determine if there is statistical difference between the probabilities of exercising the MPG under the new and the old investment regimes for workers earning three minimum wages.

Three Minimum Wages	Probability of Exercising the MPG		Abs(NR-OR)	(NR+OR)/2	Standard Error	Standard Error*1.96
	New Regime (NR)	Old Regime (OR)				
Port1	0.169	0.160	0.009	0.165	0.017	0.032
Port2	0.076	0.089	0.013	0.083	0.012	0.024
Port3	0.040	0.056	0.016	0.048	0.010	0.019
Port4	0.023	0.033	0.010	0.028	0.007	0.014
Port5	0.014	0.019	0.005	0.017	0.006	0.011
Port6	0.013	0.015	0.002	0.014	0.005	0.010
Port7	0.013	0.013	0.000	0.013	0.005	0.010
Port8	0.012	0.012	0.000	0.012	0.005	0.010
Port9	0.013	0.011	0.002	0.012	0.005	0.010
Port10	0.013	0.011	0.002	0.012	0.005	0.010

*The symbol * indicates that there is statistical difference at the 5% level of significance.*

Table E.5. Hypothesis testing to determine if there is statistical difference between the probabilities of exercising the MPG under the new and the old investment regimes for workers earning four minimum wages.

Four Minimum Wages	Probability of Exercising the MPG		Abs(NR-OR)	(NR+OR)/2	Standard Error	Standard Error*1.96
	New Regime (NR)	Old Regime (OR)				
Port1	0.006	0.006	0.000	0.006	0.003	0.007
Port2	0.004	0.003	0.001	0.004	0.003	0.005
Port3	0.003	0.001	0.002	0.002	0.002	0.004
Port4	0.003	0.001	0.002	0.002	0.002	0.004
Port5	0.003	0.001	0.002	0.002	0.002	0.004
Port6	0.003	0.001	0.002	0.002	0.002	0.004
Port7	0.003	0.001	0.002	0.002	0.002	0.004
Port8	0.002	0.001	0.001	0.002	0.002	0.003
Port9	0.001	0.001	0.000	0.001	0.001	0.003
Port10	0.002	0.001	0.001	0.002	0.002	0.003

*The symbol * indicates that there is statistical difference at the 5% level of significance.*

Table E.6. Hypothesis testing to determine if there is statistical difference between the probabilities of exercising the MPG under the new and the old investment regimes for workers earning five minimum wages.

Five Minimum Wages	Probability of Exercising the MPG		Abs(NR-OR)	(NR+OR)/2	Standard Error	Standard Error*1.96
	New Regime (NR)	Old Regime (OR)				
Port1	0.002	0.000	0.002	0.001	0.001	0.003
Port2	0.000	0.000	0.000	0.000	0.000	0.000
Port3	0.000	0.000	0.000	0.000	0.000	0.000
Port4	0.000	0.000	0.000	0.000	0.000	0.000
Port5	0.000	0.000	0.000	0.000	0.000	0.000
Port6	0.000	0.000	0.000	0.000	0.000	0.000
Port7	0.000	0.000	0.000	0.000	0.000	0.000
Port8	0.000	0.000	0.000	0.000	0.000	0.000
Port9	0.000	0.000	0.000	0.000	0.000	0.000
Port10	0.000	0.000	0.000	0.000	0.000	0.000

*The symbol * indicates that there is statistical difference at the 5% level of significance.*

Table E.7. Hypothesis testing to determine if there is statistical difference between the probabilities of exercising the MPG under the second most expensive and second cheapest commissions for workers with different wage levels who invest their balances in Port10.

Port10	Probability of Exercising the MPG		Abs(SME-SC)	(SME+SC)/2	Standard Error	Standard Error*1.96
	Second Most Expensive (SME)	Second Cheapest (SC)				
One minimum wage*	0.396	0.353	0.043	0.375	0.022	0.042
Two minimum wages	0.098	0.080	0.018	0.089	0.013	0.025
Three minimum wages	0.015	0.010	0.005	0.013	0.005	0.010
Four minimum wages	0.002	0.001	0.001	0.002	0.002	0.003
Five minimum wages	0.000	0.000	0.000	0.000	0.000	0.000
Ten minimum wages	0.000	0.000	0.000	0.000	0.000	0.000
Fifteen minimum wages	0.000	0.000	0.000	0.000	0.000	0.000
Twenty-five minimum wages	0.000	0.000	0.000	0.000	0.000	0.000

*The symbol * indicates that there is statistical difference at the 5% level of significance.*

Table E.8. Hypothesis testing to determine if there is statistical difference between the probabilities of exercising the MPG under the second most expensive and second cheapest commissions for a worker with a lifetime wage profile.

Lifetime Wage Profile	Probability of Exercising the MPG		Abs(SME-SC)	(SME+SC)/2	Standard Error	Standard Error*1.96
	Second Most Expensive (SME)	Second Cheapest (SC)				
Port1	0.003	0.001	0.002	0.002	0.002	0.004
Port2	0.001	0.000	0.001	0.001	0.001	0.002
Port3	0.000	0.000	0.000	0.000	0.000	0.000
Port4	0.000	0.000	0.000	0.000	0.000	0.000
Port5	0.000	0.000	0.000	0.000	0.000	0.000
Port6	0.000	0.000	0.000	0.000	0.000	0.000
Port7	0.000	0.000	0.000	0.000	0.000	0.000
Port8	0.000	0.000	0.000	0.000	0.000	0.000
Port9	0.000	0.000	0.000	0.000	0.000	0.000
Port10	0.000	0.000	0.000	0.000	0.000	0.000

*The symbol * indicates that there is statistical difference at the 5% level of significance.*